

February 1, 2008

UU-HEP/97-03

# Brane Creation in M(atrix) Theory

Pei-Ming Ho<sup>1</sup> and Yong-Shi Wu<sup>2</sup>

*Department of Physics, University of Utah  
Salt Lake City, Utah 84112, U.S.A.*

## Abstract

We discuss, in the context of M(atrix) theory, the creation of a membrane suspended between two longitudinal five-branes when they cross each other. It is shown that the membrane creation is closely related to the degrees of freedom in the off-diagonal blocks which are related via dualities to the chiral fermionic zero mode on a 0-8 string. In the dual system of a D0-brane and a D8-brane in type IIA theory the half-integral charges associated with the “half”-strings are found to be connected to the well-known fermion-number fractionalization in the presence of a fermionic zero mode. At sufficiently short distances, the effective potential between the two five-branes is dominated by the zero mode contribution to the vacuum energy.

---

<sup>1</sup>Address after September 1, 1997: Department of Physics, Jadwin Hall, Princeton University, Princeton, New Jersey 08544, U.S.A.

<sup>2</sup>Address after September 1, 1997: School of Natural Sciences, Institute for Advanced Study, Olden Lane, Princeton, New Jersey 08540, U.S.A.

# 1 Introduction

It was first suggested by Hanany and Witten [1] that when two branes (in appropriate configuration) cross each other, a third brane stretching between them is created or annihilated. They observed the creation in type IIB string theory of a D3-brane when an NS5-brane crosses a D5-brane. It is related to creation of branes of other dimensions by sequences of dualities [1, 2, 3, 4, 5]. In particular it is dual to the creation of a fundamental string when a D0-brane crosses a D8-brane in type IIA theory, as well as the creation of a membrane in M theory when two five-branes sharing one common dimension cross each other. In [2] it was argued from the anomaly equation that when the two branes in question cross each other an energy level crosses zero, and thus a single particle or hole is created. The creation of a particle is understood as the creation of an open string or brane. In [3] it was also shown that the induced charge on the D8-brane world-volume indicate the creation of a string when the D0-brane crosses the D8-brane.

In this paper we try to understand the phenomenon of brane creation in the context of M(atrix) theory [6]. It is most convenient to consider the creation of a longitudinal membrane when crossing two longitudinal five-branes. The classical background of the longitudinal five-branes are described by topologically nontrivial gauge field configurations residing in diagonal blocks [7]. We will show that the membrane creation is closely related to the degrees of freedom in the off-diagonal blocks. In accordance with T-duality, the equations of motion for the off-diagonal blocks in the background of two longitudinal five-branes are formally the same as the case of the 0-8 string. In a previous paper [8], the existence of the chiral fermionic zero mode on the 0-8 string has been derived, using the index theorem, from the classical equations of motion in M(atrix) theory for the off-diagonal blocks in the background of diagonal ones. It is this chiral fermionic zero mode in the off-diagonal blocks that gives rise to the energy level, which crosses zero, proposed in [2] (Sec.2). The energy of this fermionic mode is linear in the distance between the five-branes and the proportional factor equals the membrane tension so that upon quantization it can be associated with the production of membrane (Sec.3). The fact that the induced charge on the D8-brane world-volume is  $\pm 1/2$ , as a result of proper operator ordering, is closely related to the well-known fractionalization of fermion number [9] due to the existence of a fermionic zero mode or mid-gap mode (Sec.4). We will also argue that when the five-branes are sufficiently close to each other, the effective potential between them is dominated by the contribution from the off-diagonal degrees of freedom associated with this zero mode (Sec.5).

Our results agree with string theory calculations [10, 4] for the effective potential between a D0-brane and a D8-brane in type IIA theory. Our study indicates that in the M(atric) model the description of creating an open membrane stretching between two five-branes necessarily involves second quantization of degrees of freedom residing in off-diagonal blocks.

## 2 Fermionic Zero Mode in Off-Diagonal Blocks

The M(atric) theory is defined by the action [6]

$$S = \int dt \operatorname{Tr} \left( \frac{1}{2R} \dot{X}_i^2 + \frac{R}{4} (2\pi T_2^M)^2 [X_i, X_j]^2 + \frac{i}{2} \Psi^\dagger \dot{\Psi} + \frac{R}{2} (2\pi T_2^M) \bar{\Psi} \Gamma^i [X_i, \Psi] \right), \quad (1)$$

where  $i, j = 1, 2, \dots, 9$ ,  $R$  is the radius of the eleventh dimension and  $T_2^M$  is the membrane tension.

Consider in M theory two five-branes lying in directions (1,2,3,4,11) and directions (5,6,7,8,11), respectively. In M(atric) theory, the 11-th direction is the longitudinal direction for an infinite momentum frame, and the above configuration is described by matrices in the block form:

$$X_\mu = \begin{pmatrix} Z_\mu & y_\mu \\ y_\mu^\dagger & W_\mu \end{pmatrix}, \quad \Psi = \begin{pmatrix} \Theta & \theta \\ \theta^\dagger & \psi \end{pmatrix}. \quad (2)$$

The five-branes are residing in the diagonal blocks and treated as background: We take  $Z_a = 0$  for  $a = 5, \dots, 8$ ,  $W_i = 0$  for  $i = 1, \dots, 4$ ,  $Z_9 = 0$  and  $W_9 = x_9 \mathbf{1}$ , while  $Z_i$  and  $W_a$  are realized as covariant derivatives with topologically nontrivial gauge field configurations on two four-tori  $T^4$ 's [7]. (We will use  $i, j, k, \dots$  for the values 1, 2, 3, 4;  $a, b, c, \dots$  for 5, 6, 7, 8 and  $\mu, \nu, \kappa$  for 0, 1,  $\dots$ , 9.) Superpartners of  $Z_\mu$  and  $W_\mu$  ( $\Theta$  and  $\psi$ ) are set to be zeros. The coordinate  $x_9$  gives the transverse distance between five-branes. The variables  $y_\mu$  and  $\theta$  in the off-diagonal blocks, dependent on the coordinates of the torus  $T^8 = T^4 \times T^4$ , represent the matrix model degrees of freedom which are analogues of open strings between D-branes in string theory. We will treat them quantum mechanically, and their fluctuations will give rise to the interactions between the BPS branes.

By compactifying dimensions (1, 2, 3, 4, 11), one can use dualities to relate the creation of a membrane by crossing two five-branes in M theory to the creation of a string by crossing a D0-brane with a D8-brane in IIA theory. Thus we can compare our results about the former with string theory calculations for the latter.

The M(atr)ix theory action induces an action for  $y$  and its superpartner  $\theta$  in the background of  $Z_i$  and  $W_a$ . It is easy to see that in accordance with T-duality, this action for  $y$  and  $\theta$  is formally the same as that for a 0-8 string (in the strongly coupled type IIA theory) which we derived previously in [8], with the  $U(K)$  covariant derivatives<sup>3</sup>  $D_i = iZ_i$ ,  $D_a = -iW_a$  defined on a dual eight-torus  $T^8$ , except that now the distance between the D0-brane and D8-brane is  $x_9$  instead of zero. The action for  $\theta$  is

$$L_F = \int \theta^\dagger \left( i\dot{\theta} - 2\pi R T_2^M \left( i \sum_{\mu=1}^8 \Gamma^{0\mu} D_\mu + \Gamma^{09} x_9 \right) \theta \right), \quad (3)$$

which is integrated over the dual  $T^8$ .

An example for the five-brane configuration is given by  $[D_{2n-1}, D_{2n}] = -if$  with a constant  $f = 2\pi R_{2n-1} R_{2n} / K$  for  $n = 1, 2, 3, 4$  [7, 8]. When  $x_9 = 0$ , it was shown for this example [8] that there is only one chiral fermionic zero mode for  $\theta$  and no zero mode (nonvanishing classical solution) for the bosonic partner  $y$ . The zero mode solution for  $\theta$  and the spectrum of  $y$  for  $x_9 = 0$  are explicitly given in [8]. This configuration contains not only the two longitudinal five-branes but also stacks of membranes inside the five-branes. Our arguments will only rely on the existence of a chiral fermionic zero mode, so our conclusions do not depend on the details of the five-brane configurations. Since the five-brane charges for both five-branes are unity, the Chern character  $\frac{1}{4!(2\pi)^4} \int \text{Tr}(F^4)$  is one.<sup>4</sup> Hence by the index theorem [13], the difference in the number of fermionic zero modes in the two chiralities is one. In fact as there is only one zero mode on a 0-8 string [14], so by duality it must be the case that there is a single chiral fermionic zero mode for the two five-branes.

For a generic configuration of five-branes, the Hamiltonian and thus the spectrum of  $y$  depend only on  $x_9^2$  and not on  $x_9$ . But the spectrum of  $\theta$  is a little bit more complicated. The Hamiltonian for  $\theta$  is

$$H_F = 2\pi R T_2^M \int \theta^\dagger \mathcal{H} \theta, \quad (4)$$

where  $\mathcal{H} = (iD + \Gamma^{09} x_9)$  with  $D = \sum_{\mu=1}^8 \Gamma^{0\mu} D_\mu$ . The spectrum of  $\mathcal{H}^2$  is  $\{s^2 + x_9^2 \mid s \in \text{Spec}(iD)\}$ . This would imply that the spectrum of  $\mathcal{H}$  depends only on  $x_9^2$  if all  $s \neq 0$ . However, if zero is an eigenvalue of  $iD$ , then the corresponding eigenvalue(s) of  $\mathcal{H}$  can be  $\{x_9\}$ ,  $\{-x_9\}$  or  $\{x_9, -x_9\}$ . It was known [8] that when  $x_9 = 0$  there is only one zero

<sup>3</sup>The integer  $K$  is proportional to the longitudinal momentum of the five-branes [8], analogous to the length of the conjugacy class for a long string in the matrix string theory [11, 12].

<sup>4</sup>By duality this Chern character for the case of a D0-brane crossing a D8-brane is just the 8-brane charge.

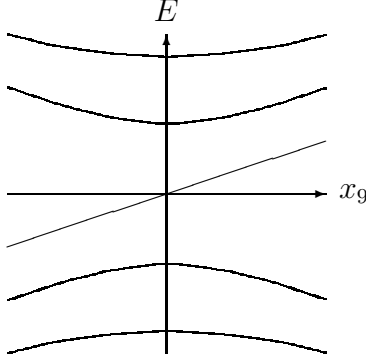


Figure 1: Spectrum of  $\theta$

mode for  $iD$ , so the last possibility is ruled out. Using the chirality of the zero mode on  $T^8$ :  $\Gamma^1\Gamma^2\cdots\Gamma^8\theta = \theta$  and its total chirality:  $\Gamma^0\Gamma^1\cdots\Gamma^9\theta = \theta$ , we find that  $\Gamma^{09}\theta = \theta$ . Thus the correct choice is  $\{x_9\}$ . This is the only part of the spectrum that depends on the sign of  $x_9$ . This property makes the fermionic zero mode behave differently from all other states in an essential way. A schematic diagram of the spectrum of  $\theta$  is in Fig.1.

When the two five-branes cross each other, the value of  $x_9$  changes its sign. While all other states are invariant under the reflection  $x_9 \rightarrow -x_9$ , the zero mode is not. This means that the two five-branes select a preferred direction in the transverse direction  $x_9$ . (By duality, this implies for the D0-D8 case that the D8-brane is oriented.) Although one can interchange the positions of the two diagonal blocks by a gauge transformation:

$$X_\mu \rightarrow UX_\mu U^\dagger, \quad \Psi \rightarrow U\Psi U^\dagger, \quad (5)$$

with

$$U = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}, \quad (6)$$

$\theta$  is interchanged with  $\theta^\dagger$  at the same time. When  $\theta$  vanishes, the exchange symmetry of the two five-branes is preserved. But when the zero mode is present, exchanging only the five-branes results in a change in the state of the system. Also, for the parity transformation  $x_9 \rightarrow -x_9$  to be a symmetry, it has to be accompanied by the change  $\Gamma^9 \rightarrow -\Gamma^9$ .

The fermionic zero mode, according to eq.(4), has the energy

$$E = 2\pi RT_2^M x_9. \quad (7)$$

If we compactify the ninth direction to a circle of radius  $R_9$ ,  $x_9$  will be promoted to  $x_9 + 2\pi n R_9$ , where  $n$  is the winding number. Then the energy of the zero mode is

$$E_n = 2\pi R T_2^M (x_9 + 2\pi R_9 n). \quad (8)$$

This is equivalent to eq.(3) given in ref.[2], where it was interpreted in the string theory context as the (fermionic) ground state energy for an open string stretching between the two branes. Here in M(atric) theory we identify it as the zero mode of fermionic off-diagonal blocks.

### 3 Second Quantization and Membrane Creation

The absolute value of the energy (7) is the same as that for a membrane stretching between two five-branes. It is therefore tempting to relate this energy to the energy for a created longitudinal membrane. For this identification to make sense it is important to notice that in M(atric) theory we are working in an infinite momentum frame, in which the energy of a longitudinal degree of freedom do not have the prefactor of  $R/N$  like the transverse degrees of freedom [7]. Due to translational invariance, its longitudinal momentum vanishes, so that  $E = M$ .

It turns out that quantization is the key to understand the membrane creation, as well as the half-charges associated with half-strings in the D0-D8 brane crossing (see Sec.4). To quantize the fermionic field  $\theta$ , as a rule we should first fill all negative-energy states, and define this (many-body) state as the vacuum of the fermion system. As for the fermionic zero mode, it can be either empty or filled. Let us denote by  $|e\rangle$  (or  $|f\rangle$ ) the state of the fermion system with all modes below the zero mode filled (all having negative energies), and with the zero mode empty (or filled). Which of them is the vacuum state depends on the sign of  $x_9$  since the zero mode energy is proportional to  $x_9$ . When  $x_9 < 0$ , the state  $|f\rangle$  is the vacuum and  $|e\rangle$  is a hole; when  $x_9 > 0$ , the state  $|e\rangle$  is the vacuum and  $|f\rangle$  is a particle. The energy of the states measured relative to the vacuum is given in Fig.2(a) and 2(b), respectively for  $|e\rangle$  and  $|f\rangle$ .

Let us denote the amplitude of the zero mode by  $\chi$ , so that  $\theta = \chi \theta^0$  where  $\theta^0$  is the zero mode solution as a function defined on  $T^8$  [8]. The canonical quantization of  $\chi$  is realized on the Hilbert space spanned by the two states  $|e\rangle$  and  $|f\rangle$ . Up to constant factors,  $\chi$  and  $\chi^\dagger$  act on the Hilbert space as annihilation and creation operators, respectively:  $\chi|e\rangle = 0$ ,  $\chi|f\rangle = |e\rangle$ ,  $\chi^\dagger|e\rangle = |f\rangle$  and  $\chi^\dagger|f\rangle = 0$ .

Whether  $\chi$  is the annihilation operator for a particle or the creation operator for a hole depends on the sign of  $x_9$ . The energies in Fig.2 are obtained by using the usual

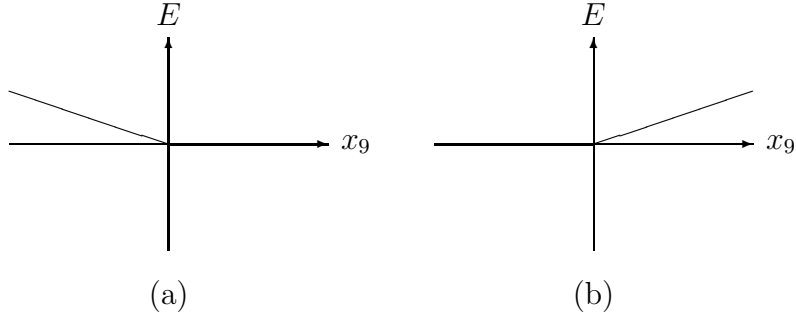


Figure 2: (a) Energy of the state  $|e\rangle$ , and (b) energy of the state  $|f\rangle$  measured relative to the vacuum

normal ordering (i.e. putting annihilation operators to the right of creation operators) for the Hamiltonian for  $x_9 < 0$  and  $x_9 > 0$  separately. Energies obtained by normal ordering are those measured relative to the ground state and thus are always non-negative.

Let us denote the vacuum as  $|0\rangle$  and the single-particle excited state as  $|1\rangle$  for all  $x_9$ . Hence  $|0\rangle$  is equal to  $|e\rangle$  for  $x_9 > 0$  but is  $|f\rangle$  for  $x_9 < 0$ . Similarly,  $|1\rangle$  is  $|f\rangle$  for  $x_9 > 0$  and is  $|e\rangle$  for  $x_9 < 0$ , corresponding to either a particle or a hole. The difference between the energies of the states  $|0\rangle$  and  $|1\rangle$  is the same as the energy of a longitudinal membrane of length  $|x_9|$ . This suggests that the states  $|0\rangle$  and  $|1\rangle$  represent the situations with zero and one open membrane stretching between the five-branes, respectively.

Now we come to the crucial point: When  $x_9$  changes adiabatically, the zero mode remains either empty or filled according to quantum adiabatic theorem. Thus, as  $x_9$  crosses zero from the positive to the negative side, from Fig.2(a) we see a spontaneous creation of a hole from the vacuum due to the spectral flow of the zero mode, while in Fig.2(b) a spontaneous annihilation of a particle into the vacuum. Everytime the energy of the fermionic zero mode crosses zero a particle or a hole is created or annihilated. The underlying physics is simply spectral flow plus filling of the Dirac sea.

Following a similar suggestion [2] in string theory, we interpret what is created or annihilated, in association with the above particle (hole) creation or annihilation, as a longitudinal membrane in the present M(atrrix) theory context. From Fig.2(a) and 2(b) one can see that for the two five-branes in question, assuming  $x_9$  is non-compact, initially there are two possibilities in the second quantized theory, either there is none or there is one longitudinal membrane stretching between them. In the first (or the

second) case, the crossing of the two five-branes will lead to creation (or annihilation) of such a membrane. The necessity of having both possibilities was argued in ref.[1] (Fig.9).

## 4 Half Fermion Number and Operator Ordering

In this section, we pay attention to the problem of the so-called half-strings in the D0-D8 case [3, 4]. We will see that this problem is related to the well-known fermion number fractionalization in the presence of a fermionic zero mode (or in general mid-gap state) [9].

Classically, the conserved fermion number for the zero mode is  $\chi^\dagger \chi$ . In quantum mechanics, the operators have to be properly ordered so that it transforms correctly under the fermion-number conjugation [9]. For an ordinary fermionic field the number operator is

$$(\sum_{\alpha} b_{\alpha}^{\dagger} b_{\alpha} - \sum_{\alpha} d_{\alpha}^{\dagger} d_{\alpha}), \quad (9)$$

where the  $b_{\alpha}$ 's and  $d_{\alpha}$ 's are the annihilation operators for particles and holes, respectively. Since the creation of a hole is the same as the annihilation of a particle in the Dirac sea, the roles of the  $b_{\alpha}$ 's and  $d_{\alpha}$ 's are symmetric up to a flip of sign in the fermion number. At  $x_9 = 0$ , the two states  $|e\rangle$  and  $|f\rangle$  are degenerate. To preserve the symmetric roles of  $|e\rangle$  and  $|f\rangle$ ,<sup>5</sup> the fermion numbers for these states should be  $\pm 1/2$  [9]. It is thus fixed up to a sign to be

$$N_F = \frac{1}{2}(\chi^\dagger \chi - \chi \chi^\dagger). \quad (10)$$

This operator only takes values of  $\pm 1/2$  (Fig.3). It is  $-1/2$  for  $|e\rangle$  and  $1/2$  for  $|f\rangle$ . The value of  $N_F$  for the vacuum changes by one when  $x_9$  crosses zero, showing the need for the creation of a membrane in order to maintain charge conservation.

Comparing with the dual IIB system of NS5-brane and D5-brane [1], the charge (10) is the total magnetic charge. The charge of the vacuum  $|0\rangle$  corresponds to the induced charge due to one of the five-branes on the other five-brane. Due to the jump of the induced charge, conservation of the total magnetic charge requires the creation of a membrane.

In [3] it was shown that in IIA theory the induced charge on a D8-brane by a D0-brane is one half and it jumps to minus one half when the branes cross. (See [1] for the

---

<sup>5</sup>Note that it is a matter of convention to say which state is empty or filled, as well as which operator is the creation or annihilation operator.



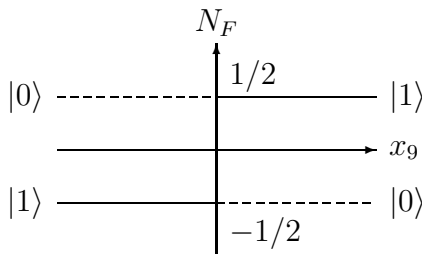


Figure 3: Fermion number operator  $N_F$  for the zero mode

IIB analogue.) The charge one half was associated with half a string [3]. By duality we see that this peculiar appearance of half a string is simply originated from the ordering of operators in quantum mechanics. To use the term half a string is in a sense just like saying that the ground state energy for a simple harmonic oscillator is due to half a quantum.

As mentioned in Sec.2, interchanging the two five-branes is accompanied by interchanging  $\theta$  and  $\theta^\dagger$ , which implies that  $N_F \rightarrow -N_F$ . Thus if we say that  $N_F$  is the induced charge on the first five-brane, the induced charge on the second five-brane would be  $-N_F$ .

In fact, the fermion number for  $|0\rangle$  jumps by one when  $x_9$  crosses zero for an arbitrary ordering of operators. When the 9-th direction is compactified,  $x_9$  becomes a  $U(1)$  gauge field in a 1+1 dimensional theory, and  $\chi$  becomes a fermionic field charged with respect to this gauge field. The jump in the fermion number causes a change in the number of charged fields and thus affects the  $U(1)$  anomaly for the 1+1 dimensional field theory [2]. In an adiabatic process where  $x_9$  passes zero, a membrane is created so that the total charge is conserved. The orientation of the membrane is determined by the charge conservation.

## 5 Energy of Zero Mode and Effective Potential

Knowing the spectrum of the zero mode (Fig.1), we can derive the effective potential in the Hamiltonian approach without much effort. The calculation for the effective potential is analogous to that for the Casimir effect. For effective potential between two branes, we are concerned about the change in the vacuum energy as  $x_9$  varies. For sufficiently small  $x_9$ , the dominating contribution to the force between the two

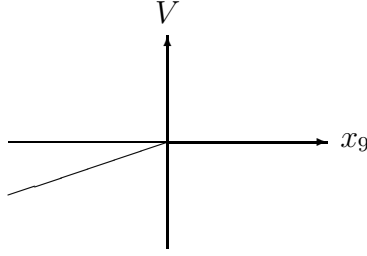


Figure 4: The effective potential between the two five-branes

branes comes from the zero mode, which is constant with respect to  $x_9$ . The reason is that except for the zero mode, the spectra of other modes of  $y$  and  $\theta$  depend only on  $x_9^2$ , which always give a force proportional to  $x_9$ . The effective potential between the five-branes is therefore approximately the energy of the zero mode.

After one integrates over  $T^8$  the Hamiltonian operator (4) becomes

$$H_F = 2\pi R T_2^M x_9 \chi^\dagger \chi. \quad (11)$$

It follows that the effective potential between the two five-branes when there is no membrane stretching between them (corresponding to the state  $|0\rangle$ ) is given by Fig.4. Since the state  $|0\rangle$  is the ground state for all  $x_9$ , it should be the state corresponding to the string theory calculations in [10, 4].

The operator ordering in (11) agrees with the normal ordering when  $x_9 > 0$  but is different from the normal ordering for  $x_9 < 0$ . Normal ordering is used to calculate the excitation energy relative to the vacuum, but here we are interested in the variation in the vacuum energy itself. One may wonder if one can choose another operator ordering, for instance, to use  $\frac{1}{2}(\chi^\dagger \chi - \chi \chi^\dagger)$  instead of  $\chi^\dagger \chi$  in (11). This ambiguity of operator ordering is fixed by requiring the exchange symmetry mentioned in Sec.2. When the zero mode is absent, exchanging the five-branes ( $Z \leftrightarrow W$ ) together with a translation in  $x_9$  results in the transformation  $x_9 \rightarrow -x_9$ . As a symmetry this should leave the energy invariant, so we should have  $H_F = 0$  for the state  $|e\rangle$ . Note that the ordering should be independent of  $x_9$  in order to exhibit the  $x_9$ -dependence of the vacuum energy.

Consider the vacuum state  $|0\rangle$ . When  $x_9 < 0$ , the zero mode is filled and the variation of its energy with respect to  $x_9$  gives a repulsive force between the five-branes. For  $x_9 > 0$ , the zero mode is empty and the force vanishes. Comparing the energy for  $|0\rangle$  and the effective potential between D0 and D8-branes by duality, we find that indeed  $|0\rangle$  is the state corresponding to the string calculations in [10, 4]. Note

that, contrary to what is suggested in some of the literature, we believe that the change in force for the state  $|0\rangle$  when  $x_9$  changes sign is not a signal of membrane creation, rather it is only a result of closed string R-R exchange. Indeed, the membrane creation is associated with the jump from  $|0\rangle$  to  $|1\rangle$ .

Combining Fig.2(a) and Fig.4, one easily sees that with the created brane included, the net potential (and force) between the two five-branes remains zero when  $x_9$  changes from negative to positive, if initially there is no membrane suspended between them.

String theory calculations [10, 4] show that the effective potential between a D0-brane and a D8-brane is

$$V = -\frac{1}{2}T_s(1 \mp 1)|x_9|, \quad (12)$$

in agreement with our results. The first term above comes from the traces over the NS and R sectors and the second term from the  $R(-1)^F$  sector of open strings. The sign depends on whether the D0-brane is on the left or right of the D8-brane. Thus the sign flips when the branes cross. This is identical to what we see in the M(atrrix) theory. When  $x_9$  crosses zero for the state  $|0\rangle$ , only the zero mode will change its fermion number by one (see Fig.3) and cause a change in sign of the  $R(-1)^F$  sector. The string theory calculation also shows that one has to attribute part of the effective potential to gravitons and dilatons so that the rest is due to the contribution of half a string. In M(atrrix) theory we no longer distinguish contributions from the NS, R or  $R(-1)^F$  sectors,<sup>6</sup> but we can understand interactions solely in terms of off-diagonal blocks. While half a string can hardly be physical in string theory, we now have a better understanding of the total effect of interactions in M(atrrix) theory.

The low energy effective potential between the five-branes can also be calculated by integrating out  $y$  and  $\theta$ . Since the action for  $y$  and  $\theta$  is the same as the one for 0-8 strings (in the strong coupling limit), we can apply the results of [15]. Though it was claimed in [15] that the M(atrrix) theory result does not agree with the string theory calculation [15, 10], we note that this discrepancy is due to an assumption that the potential is independent of the sign of  $x_9$ . Without this assumption the calculation in [15] would have given a result consistent with our calculation of the vacuum energy.

## 6 Remarks

We conclude this paper by a few remarks.

---

<sup>6</sup>However, it appears that the two terms in (12) correspond to the decomposition of  $\chi^\dagger\chi$  in (11) as  $\frac{1}{2}(\chi^\dagger\chi + \chi\chi^\dagger) + \frac{1}{2}(\chi^\dagger\chi - \chi\chi^\dagger)$  for the state  $|0\rangle$ .

1. We find it much easier to calculate the effective potential and to understand the creation of an open membrane in M(atrix) theory than in string theory. What is essential to our arguments in the M(atrix) theory is the generic feature of the spectrum (Fig.1) for the off-diagonal blocks. The quantization of the fermionic zero mode associates the creation of the open membrane in its ground state with the creation and annihilation operators  $\chi^\dagger, \chi$ . The notion of “half a membrane” or “half a string” is understood as a result of operator ordering appropriate in the presence of the fermionic zero mode.
2. We used the zero mode in the off-diagonal blocks to describe the creation of an open membrane, while the five-branes are given by the diagonal blocks. This is to be contrasted with other descriptions of open membranes by using  $SO(N)$  matrices [16] or by modifying the closed membrane configuration [17].
3. The intriguing behavior of the zero mode is due to the fact that the zero mode is chiral in the transverse direction. One may apply similar arguments to the generic case of a  $Dp$ -brane and a  $Dp'$ -brane. But except those dual to the five-branes discussed above, there is no similar phenomenon because the fermionic zero modes of both chiralities in the transverse direction are paired.
4. Applying our discussions to the case of one D0-brane in the presence of two D8-branes considered in [4], we find for the ground state that the forces in the three regions divided by the two D8-branes are from left to right  $-2T_s$ ,  $-T_s$  and 0, respectively. It is simply the superposition of the forces on the D0-brane due to the two D8-branes. However in [4] it was argued that the forces from left to right should be  $-2T_s$ , 0 and 0, respectively. In their arguments they used the cancellation between two half-strings associated with the two D8-branes as the total induced charge vanishes when the D0-brane lies between the two D8-branes. However in M(atrix) theory the five-branes are naturally associated with Chan-Paton factors, thus the two half-strings with different Chan-Paton factors can not cancel.

## 7 Acknowledgment

We thank Igor Klebanov for helpful comments. Y.S.W. thanks Japan Society for the Promotion of Sciences for an Invitational Fellowship and Institute for Solid State

Physics, University of Tokyo for warm hospitality, where part of work was done. This work is supported in part by U.S. NSF grant PHY-9601277.

## References

- [1] A. Hanany, E. Witten: “Type-II B Superstrings, BPS Monopoles and Three-Dimensional Gauge Dynamics”, hep-th/9611230.
- [2] C. P. Bachas, M. R. Douglas, M. B. Green: “Anomalous Creation of Branes”, hep-th/9705074.
- [3] U. Danielsson, G. Ferretti, I. R. Klebanov: “Creation of Fundamental Strings by Crossing D-Branes”, hep-th/9705084.
- [4] O. Bergman, M. R. Gaberdiel, G. Lifschytz: “Branes, Orientifolds and the Creation of Elementary Strings”, hep-th/9705130.
- [5] S. P. de Alwis: “A note on brane creation”, hep-th/9706142.
- [6] T. Banks, W. Fischler, S. H. Shenker, L. Susskind: “M Theory as a Matrix Model: A Conjecture”, *Phys. Rev.* **D55**, 5112-5128 (1997); hep-th/9610043.
- [7] T. Banks, N. Seiberg, S. Shenker: “Branes from Matrices”, *Nucl. Phys.* **B490**, 91-106 (1997); hep-th/9612157.
- [8] P.-M. Ho, M. Li, Y.-S. Wu: “ $p$ - $p'$  Strings in M(atric) Theory”, hep-th/9706073.
- [9] R. Jackiw, C. Rebbi: “Solitons with Fermion Number  $1/2$ ”, *Phys. Rev.* **D13** (1976) 3398-3409.
- [10] G. Lifschytz: “Comparing D-Branes to Black Branes”, *Phys. Lett.* **B388** (1996) 720-726; hep-th/9604156.
- [11] T. Banks, N. Seiberg: “Strings from Matrices”, hep-th/9702187.
- [12] R. Dijkgraaf, E. Verlinde, H. Verlinde: “Matrix String Theory”, hep-th/9703030.
- [13] M. F. Atiyah, I. M. Singer: *Ann. Math.* **87**, 485 (1968); *Ann. Math.* **87**, 546 (1968);  
M. F. Atiyah, G. B. Segal: *Ann. Math.* **87**, 531 (1968).

See, for example, M. Nakahara: *Geometry, Topology and Physics*, Institute of Physics Publishing (1990) for an introduction.

- [14] J. Polchinski: “TASI Lectures on D-branes”, hep-th/9611050.
- [15] J. M. Pierre: “Interactions of Eight-Branes in String Theory and M(atrix) Theory”, hep-th/9705110.
- [16] D. B. Fairlie, P. Fletcher, C. K. Zachos: “Trigonometric Structure Constants For New Infinite-Dimensional Algebras”, *Phys. Lett.* **218B** (1989) 203-206;  
C. N. Pope, L. J. Romans: “Local Area-Preserving Algebras For 2-Dimensional Surfaces”, *Class. Quant. Grav.* **7** (1990) 97-109;  
N. Kim, S.-J. Rey: “M(atrix) Theory on an Orbifold and Twisted Membrane”, hep-th/9701139.
- [17] M. Li: “Open Membranes in Matrix Theory”, *Phys. Lett.* **B397** (1997) 37-41; hep-th/9612144.